Name:______________________________________**Date:**_______________________

Gettin' On Down to One

This problem is about a certain pair of rules for generating a sequence of numbers. You can start with any positive whole number as the first term. After that, you find each term by applying one of the two rules to the current term. The decision about which rule to apply depends on whether the current term is an odd number or an even number.

- If the current term is an ***odd*** number, the rule is
  Multiply the current term by 3 and then add 1 to get the next term in the sequence.
- If the current term is an ***even*** number, the rule is
  Divide the current term by 2 to get the next term in the sequence.

For example, suppose the starting number is 7. Since this is an odd number, you use the first rule: multiply by 3 then add 1. That gives 22, so the second term in the sequence is 22. Since 22 is even, you now use the second rule: divide by 2. That gives 11, so the third term in the sequence is 11. And so on.

Following the correct rule each time, starting with 7, you generate the sequence

7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1, . . .

Notice that any time this procedure gets to 1, the sequence will then go to 4, 2, 1 again, over and over. So you should consider the sequence to be finished if it reaches 1.

In the case above, where you started with 7, it took 16 steps to reach 1. (The starting number itself is not counted as a step.)

1. Use the pair of rules above to generate and record sequences for each of these starting numbers

a) 6 b) 9 c) 21 d) 33

2. In Questions 1a through 1d, you should have found that each sequence eventually reached 1. Find out, for each case, how many steps it took to reach 1.

3. The only starting number that gets to 1 in only one step is 2, and the only starting number that gets to 1 in only two steps is 4.

Which starting numbers will get down to 1 after only three steps? Four steps? Five steps? Explore.

4. Describe a way to find starting numbers that will produce very long sequences, such as 100 steps.

5. What other observations can you make about how this procedure works?